Regression Verification: Proving Partial Equivalence

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Seminar within the

Projektgruppe Formale Methoden der Softwareentwicklung

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Introduction

Formal Verification

Formally prove correctness of software
$\Rightarrow$ Requires formal specification

Regression Testing

Discover new bugs by testing for them
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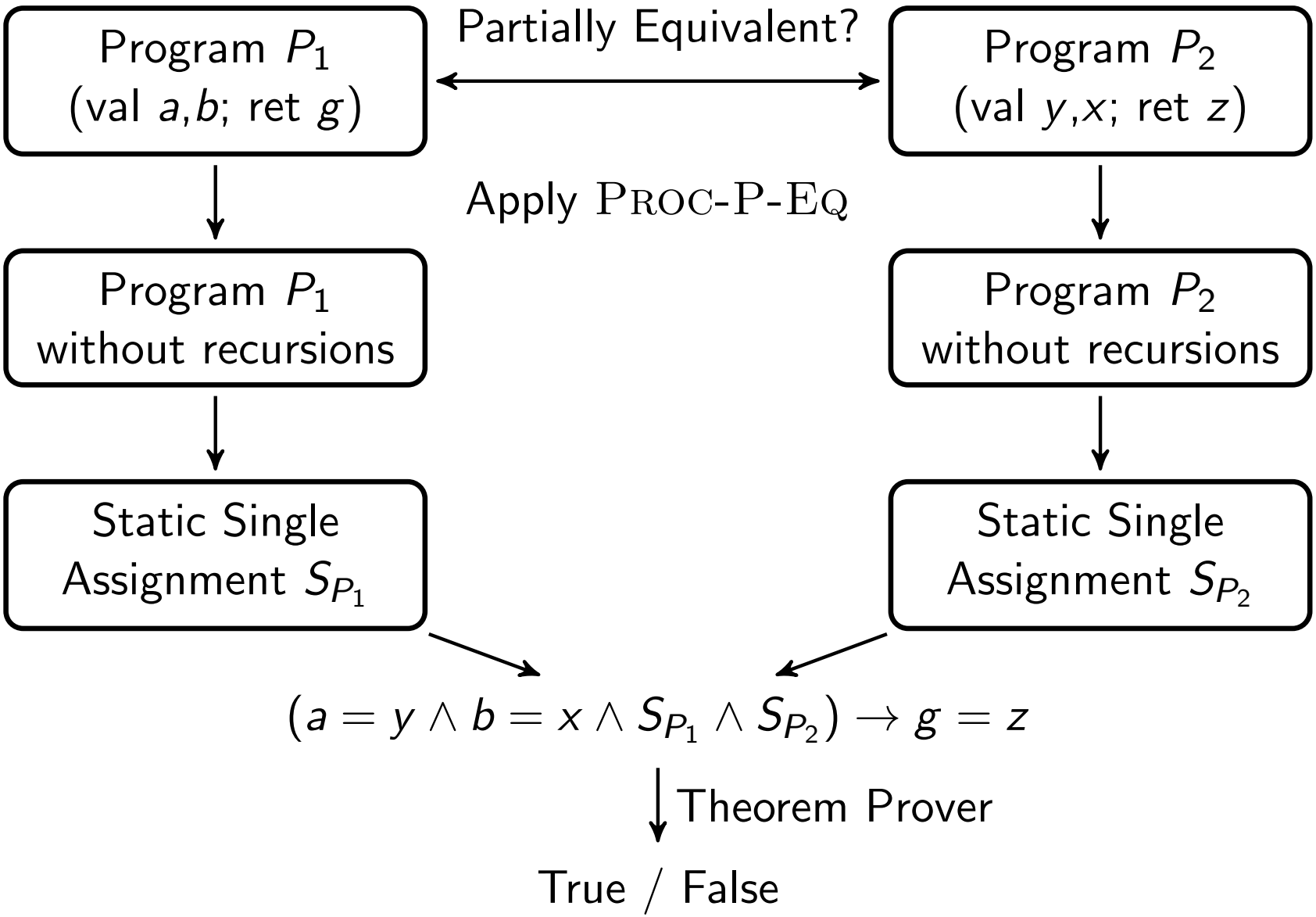
- Goal: Proving the equivalence of two closely related programs
- No formal specification or test cases required
- Instead use old program version
- Make use of similarity between programs

Overview

1. Theoretical Framework
2. Practical Framework
3. Limitations

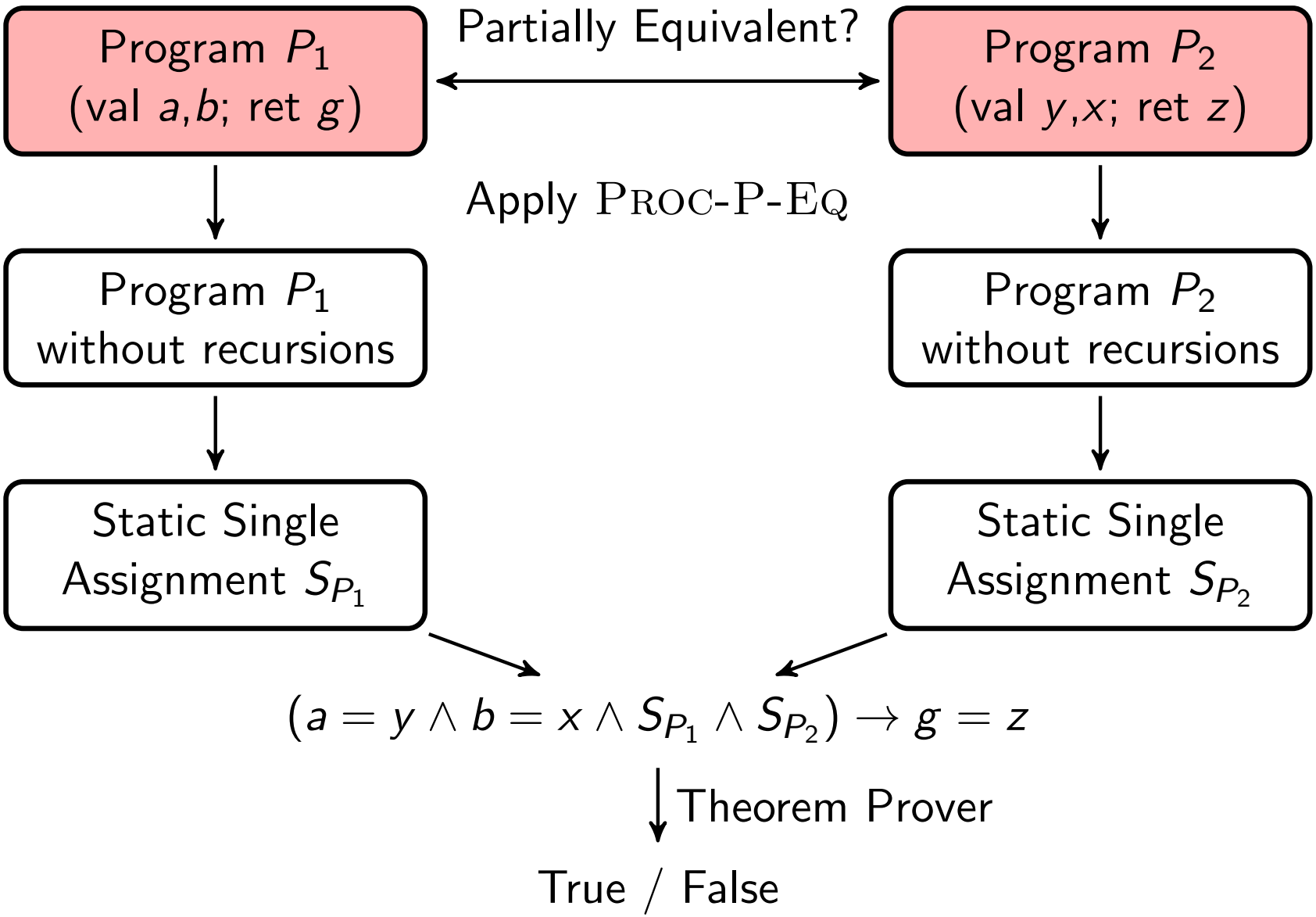
Theoretical Framework

Overview



Linear Procedure Language

Overview



Linear Procedure Language

Example

```
procedure gcd3(val x,y,z; ret w):
  call gcd(x,y; a);
  call gcd(a,z; w);
  return

procedure gcd(val a,b; ret g):
  if b = 0 then
    g := a
  else
    a := a%b;
    call gcd(b,a; g)
  fi;
  return
```

Linear Procedure Language

Syntax

$$
\begin{array}{rcl}
\mathit{Program} & :: & \langle \textbf{procedure p}(\textbf{val } \overline{arg - r_p}; \textbf{ ret } \overline{arg - w_p}) : S_p \rangle_{p \in Proc} \\
S & :: & x := \mathrm{e} \\
 & | & \mathrm{S} \ ; \ \mathrm{S} \\
 & | & \textbf{if } \mathrm{B} \textbf{ then } \mathrm{S} \textbf{ else } \mathrm{S} \textbf{ fi} \\
 & | & \textbf{if } \mathrm{B} \textbf{ then } \mathrm{S} \textbf{ fi} \\
 & | & \textbf{call } p(\overline{e}; \overline{x}) \\
 & | & \textbf{return}
\end{array}
$$

$\Rightarrow$ No loops

Partial Equivalence

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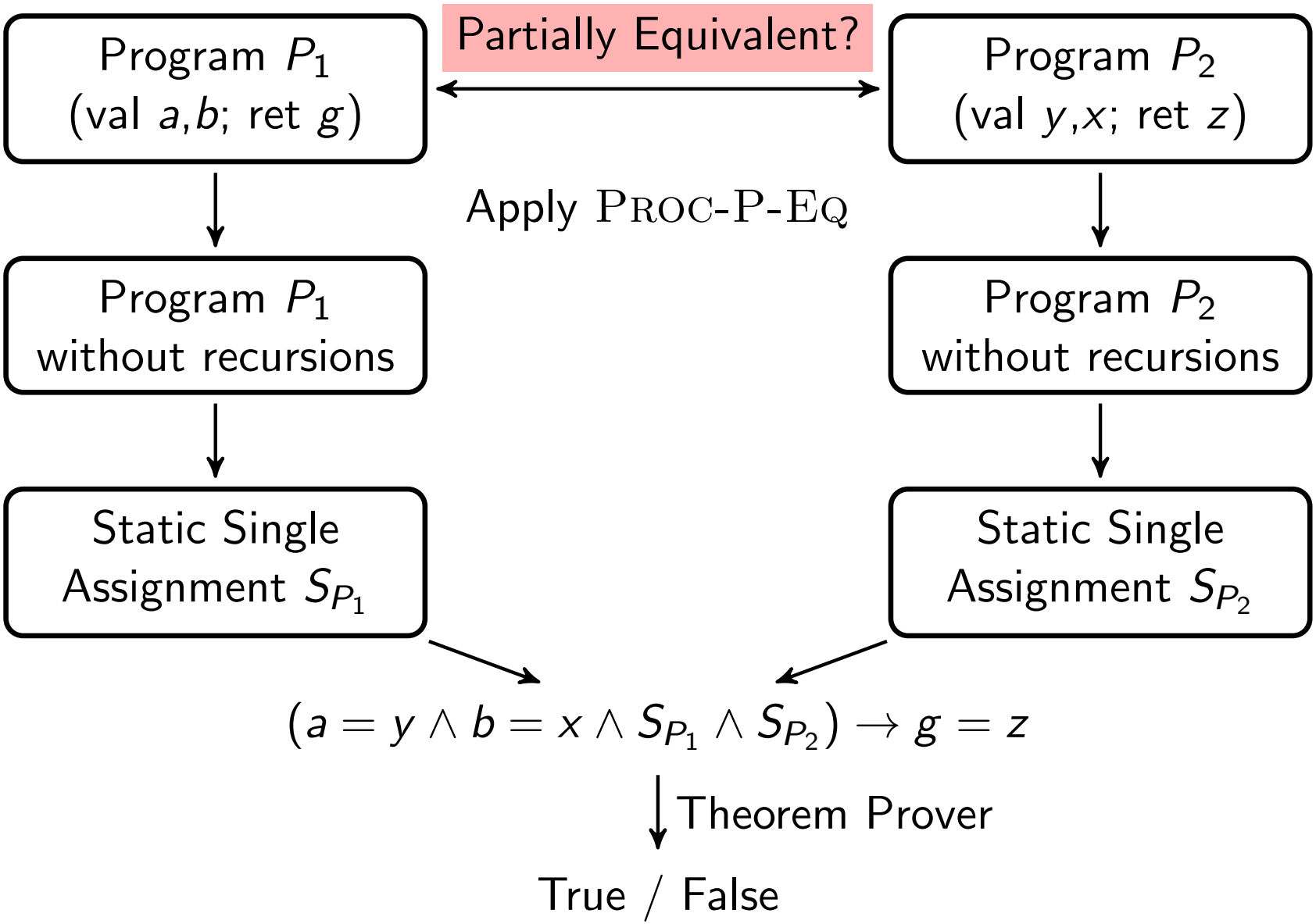


Partial Equivalence

Partial Equivalence: Given the same inputs, any two terminating executions of programs P_1 and P_2 return the same value.

$\Rightarrow$ Partial Equivalence is undecidable

In LPL:

$$\text{part-equiv}(P_1, P_2) = in[P_1] = in[P_2] \rightarrow out[P_1] = out[P_2]$$

Uninterpreted Procedures

Given the same inputs an Uninterpreted Procedure always produces the same outputs.

In LPL:

```
procedure U(val r1,r2,...; ret w1,w2,...):
  return
```

Mappings

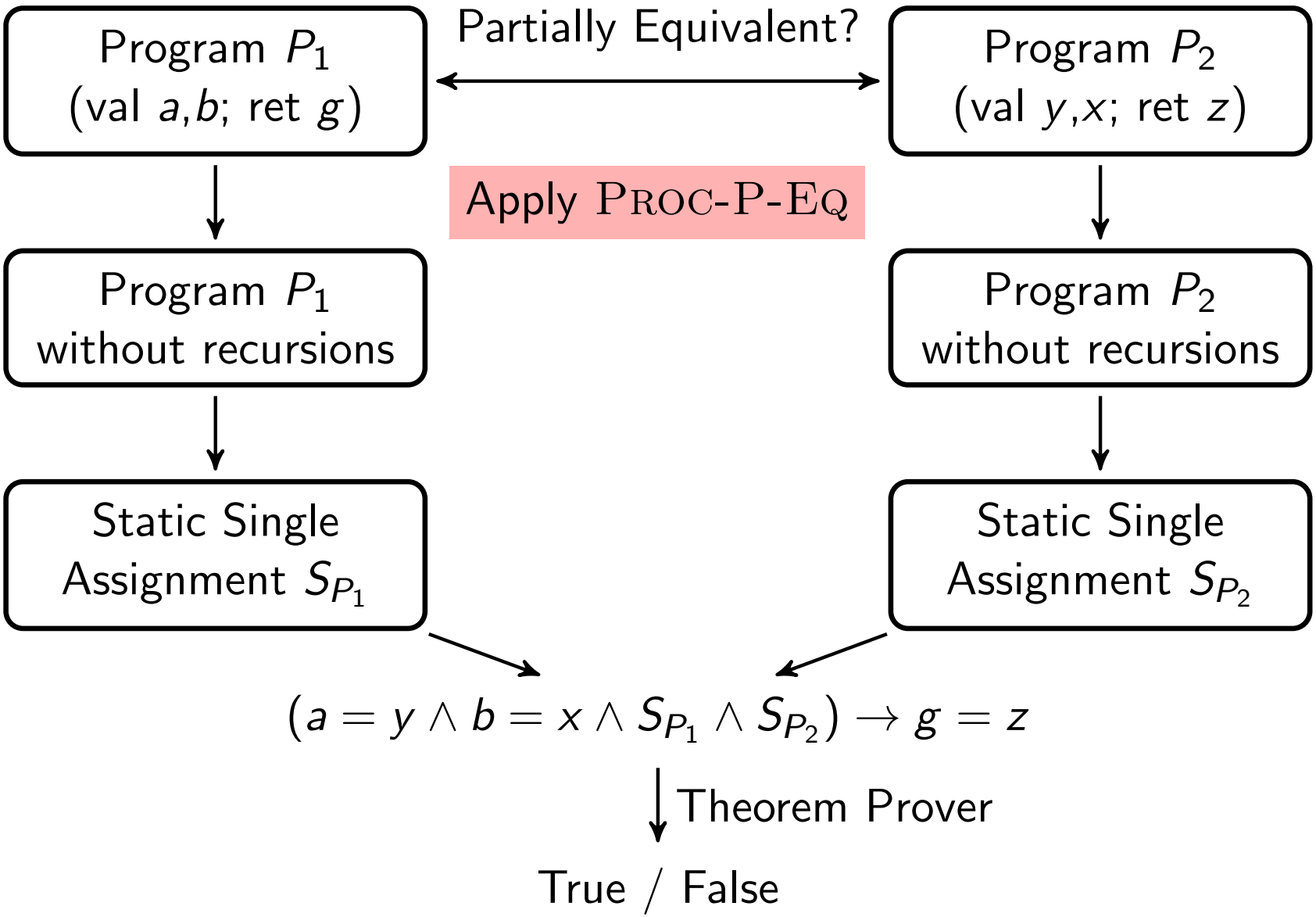
Programs P_1 and P_2 consist of procedures
Map equivalent procedures onto each other

In LPL:
$map : Proc[P_1] \mapsto Proc[P_2]$

UP maps procedures to their respective uninterpreted procedures:

$$\langle F, G \rangle \in map \iff UP(F) = UP(G)$$

Rule for Proving Partial Equivalence



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$$\frac{\text{part-equiv(gcd1, gcd2)} \vdash \text{part-equiv}(\mathit{gcd}1\ \textbf{body}, \mathit{gcd}2\ \textbf{body})}{\text{part-equiv}(\mathit{gcd}1, \mathit{gcd}2)}$$

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procedure gcd1
(val a,b; ret g):
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procedure gcd2
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Rule Proc-P-Eq

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- $\mathbb{L}_{\mathbb{UP}}$ is a sound proof system for a non-recursive LPL
- $F^{UP} = F[f \leftarrow UP(f) \mid f \in Proc[P]]$ is an isolated procedure

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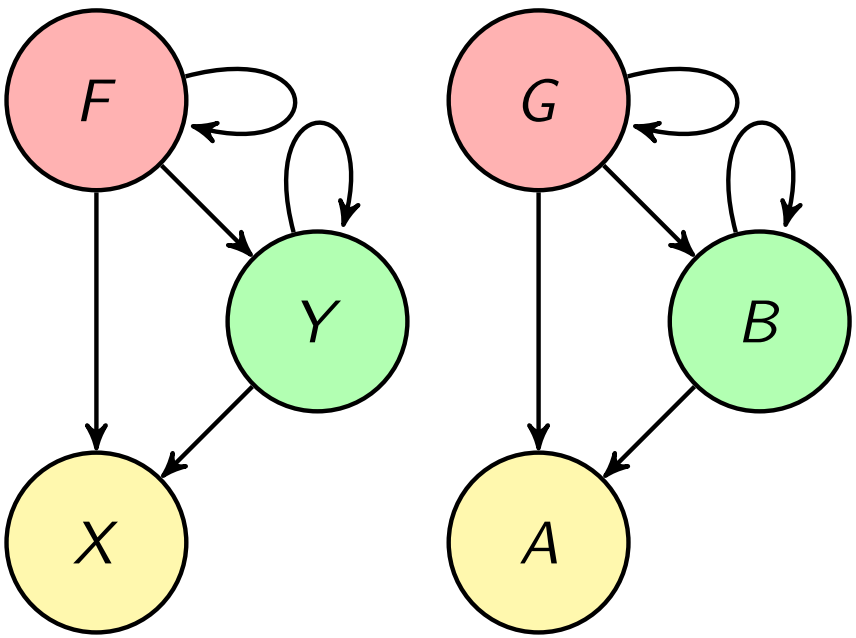

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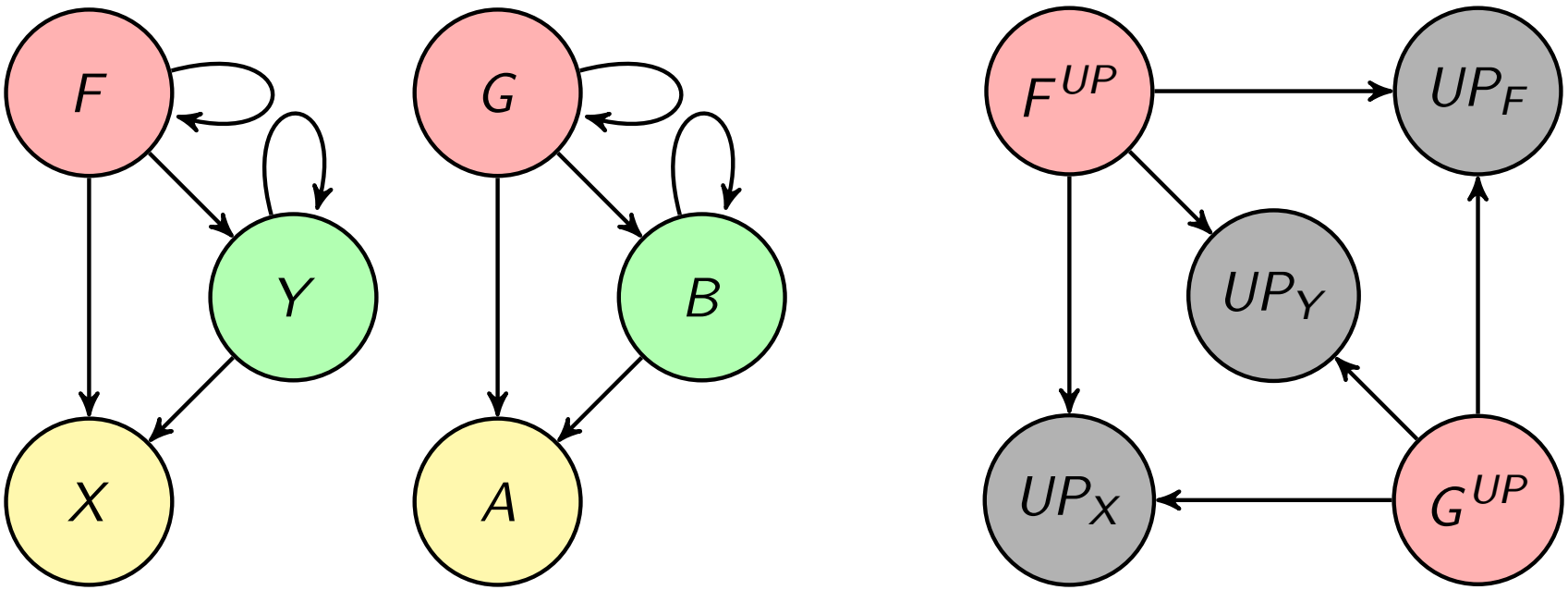

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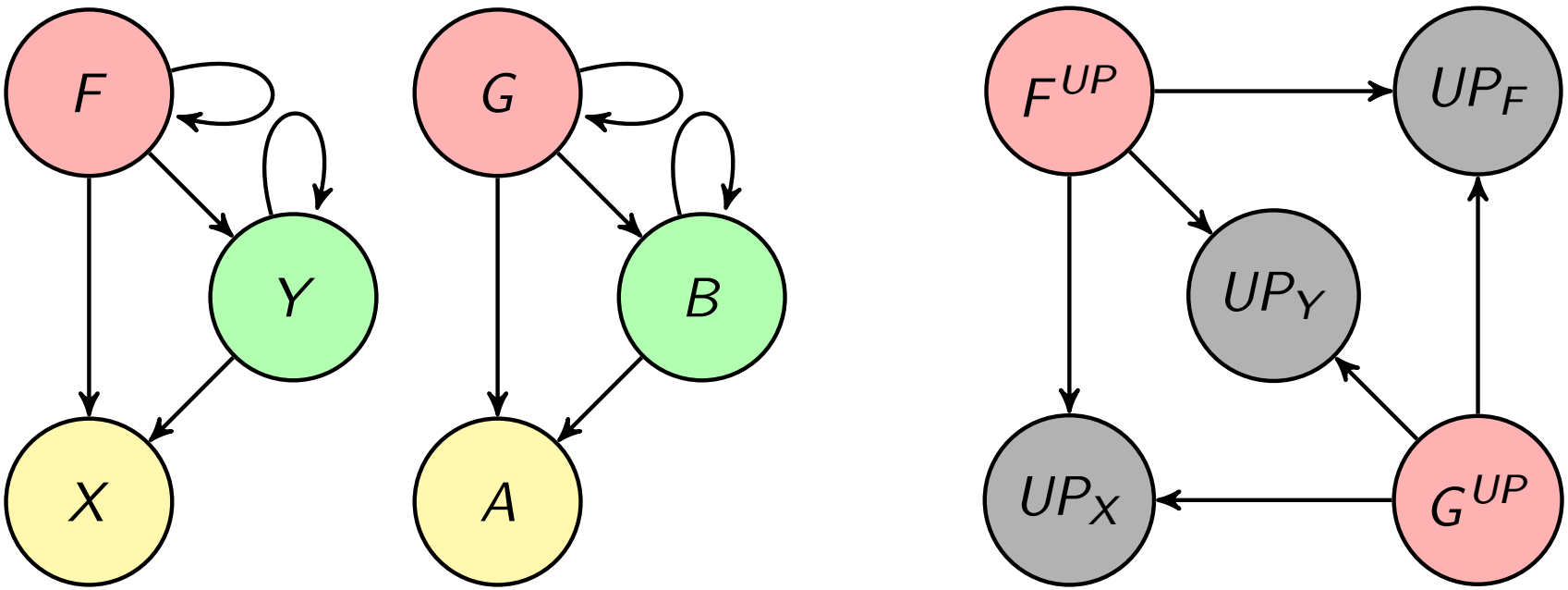

$\Rightarrow$ Proc-P-Eq is sound, not complete

Static Single Assignment

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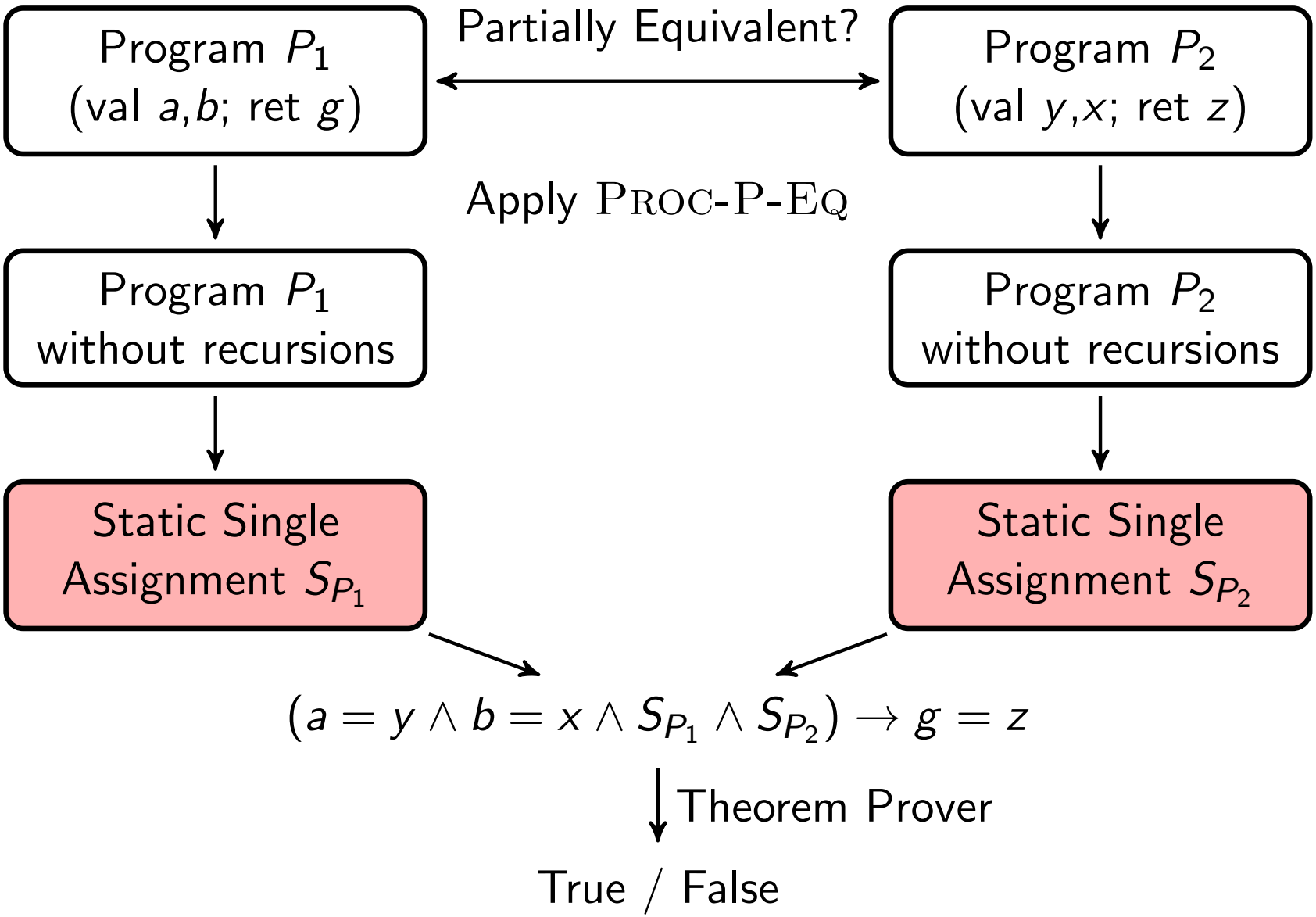

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Formula

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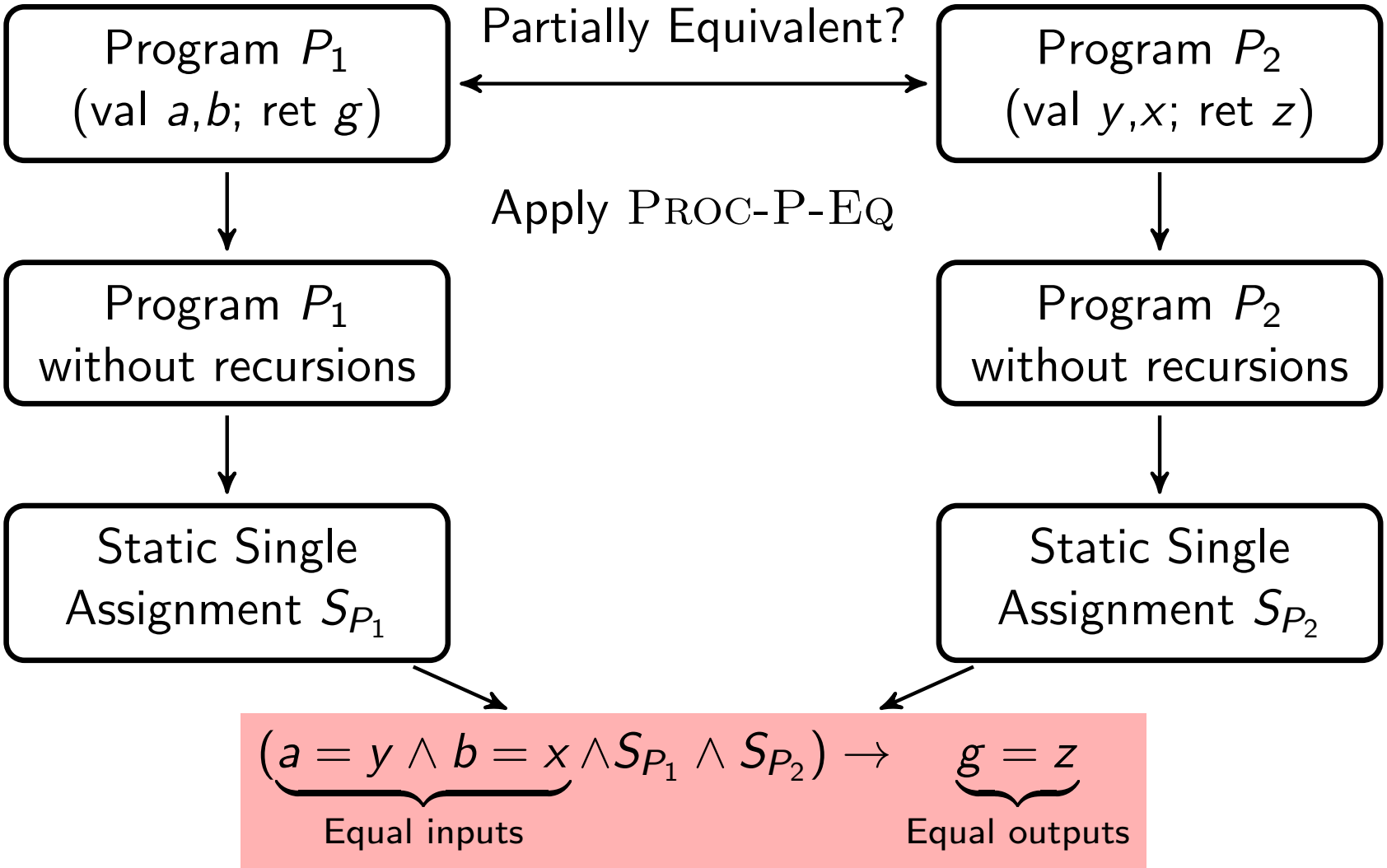

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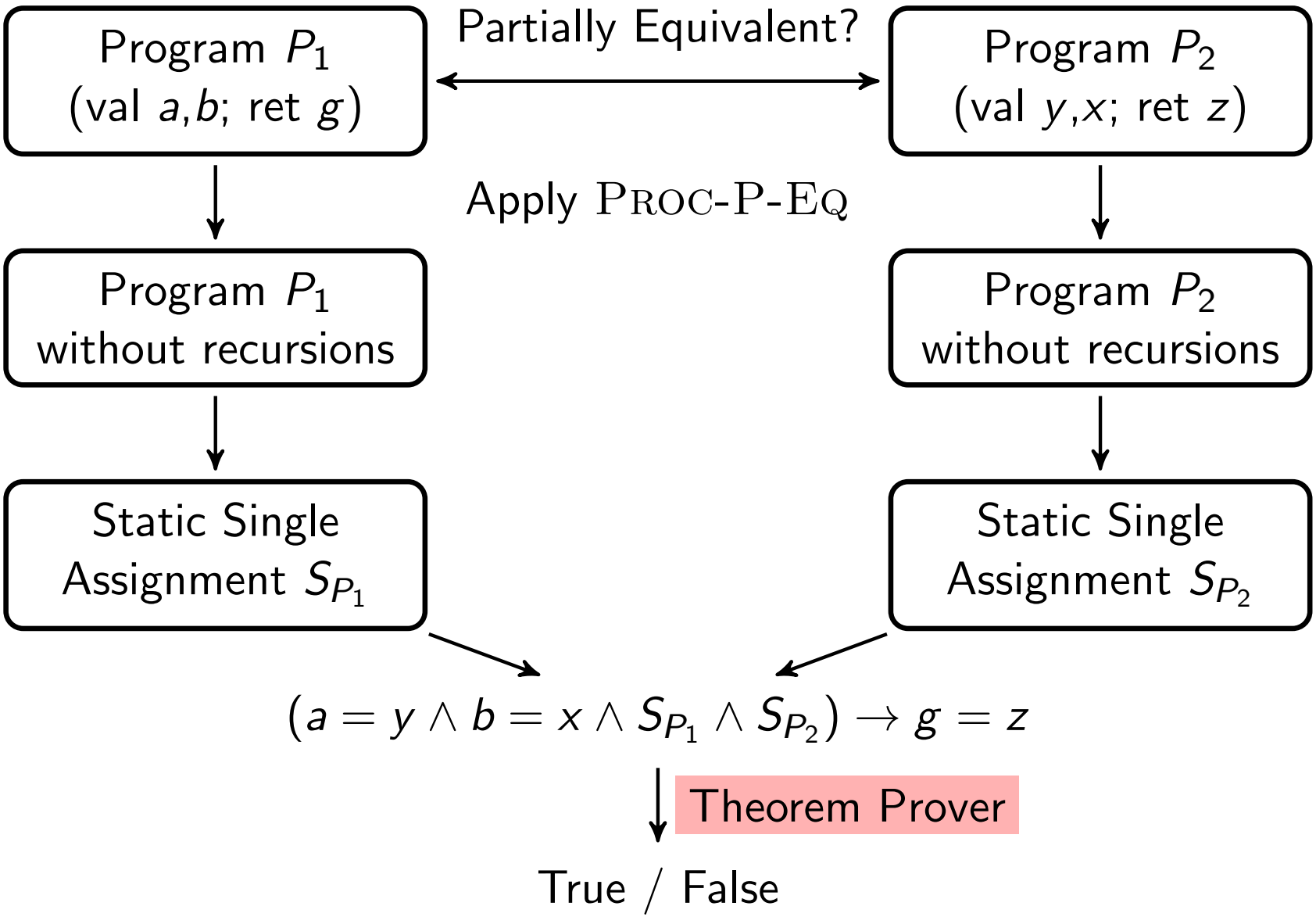

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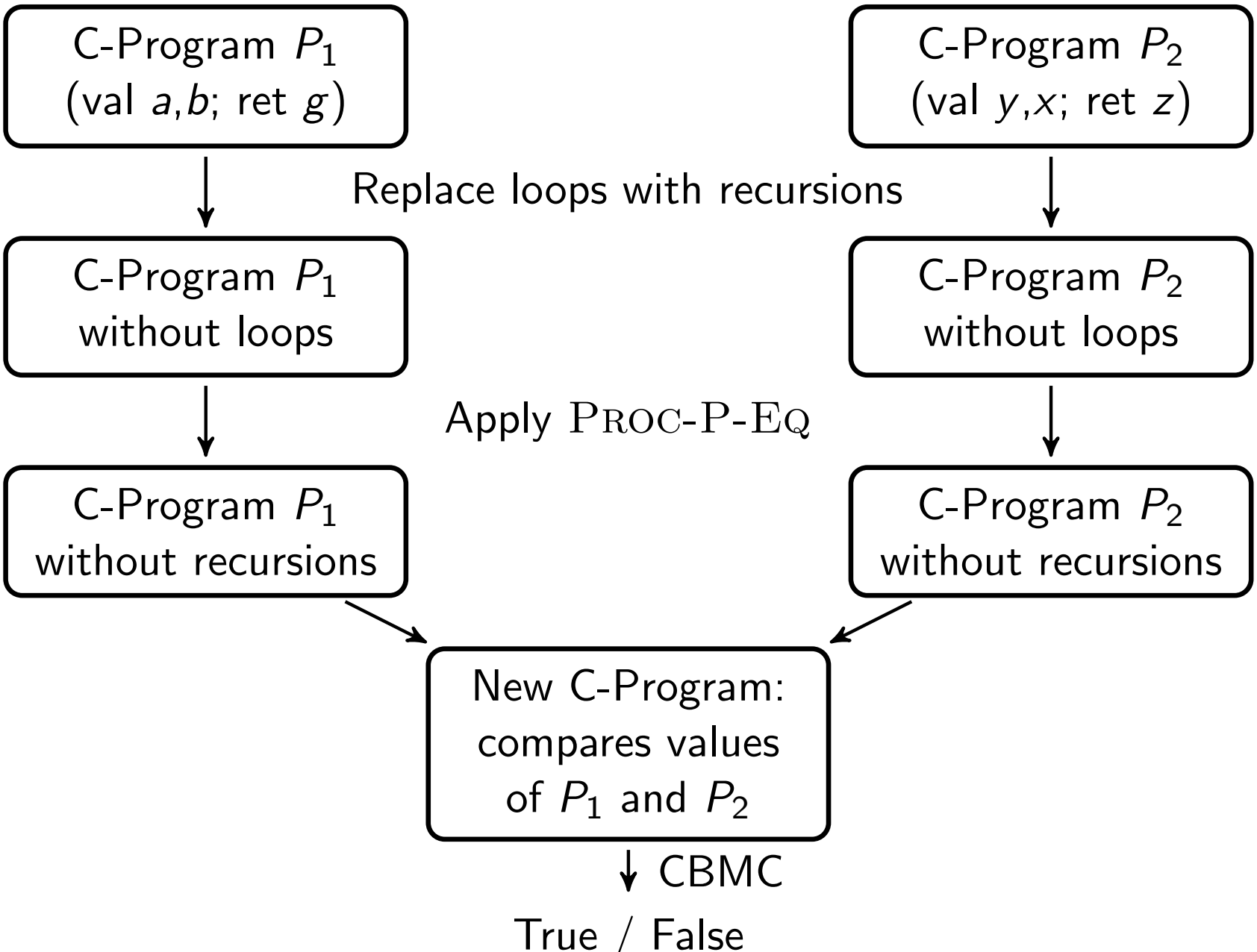
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C-Program P_2
(val y,x; ret z)
Replace loops with recursions
C-Program P_1
without loops
C-Program P_2
without loops
Apply PROC-P-EQ
C-Program P_1
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C-Program P_2
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New C-Program:
compares values
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CBMC
True / False

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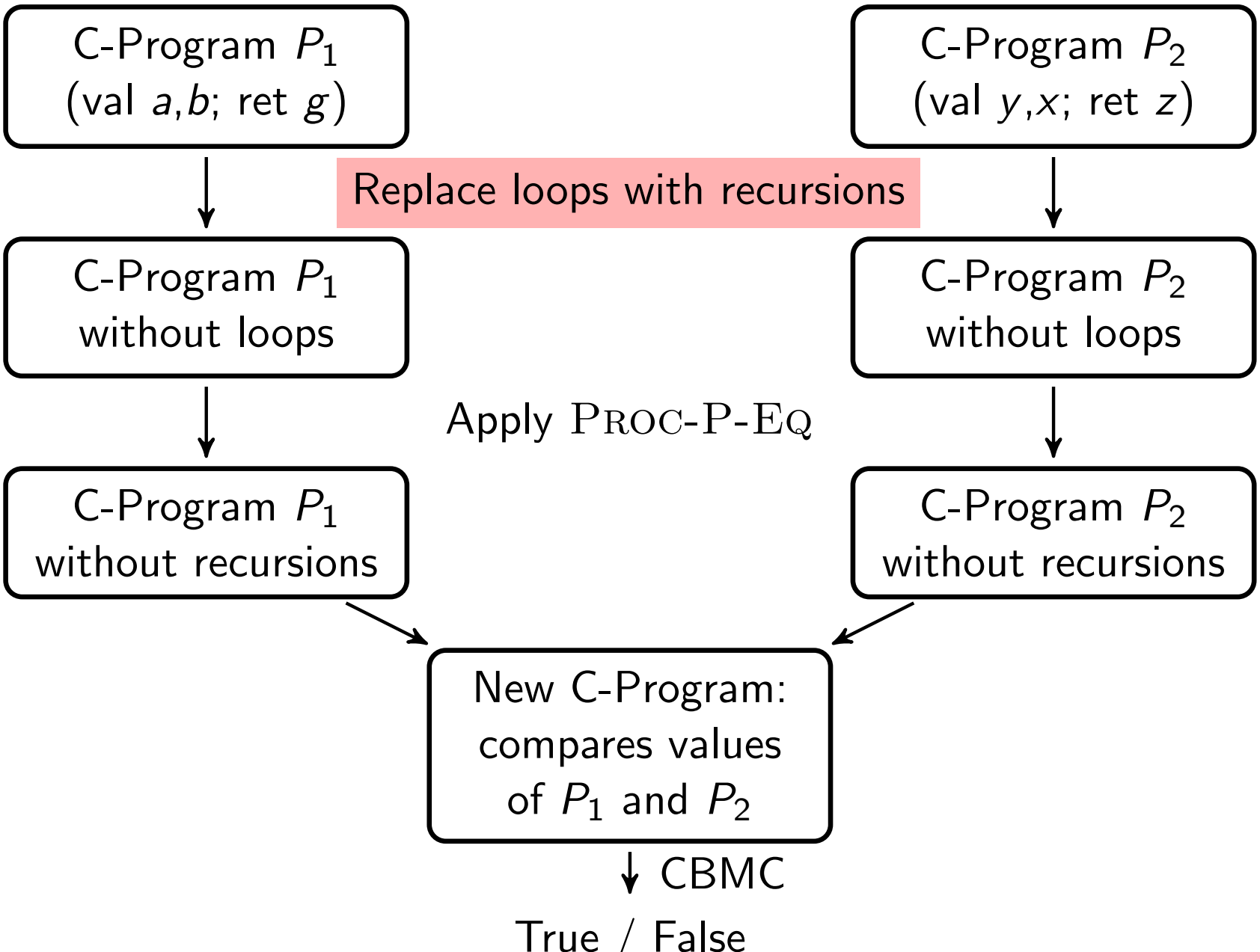
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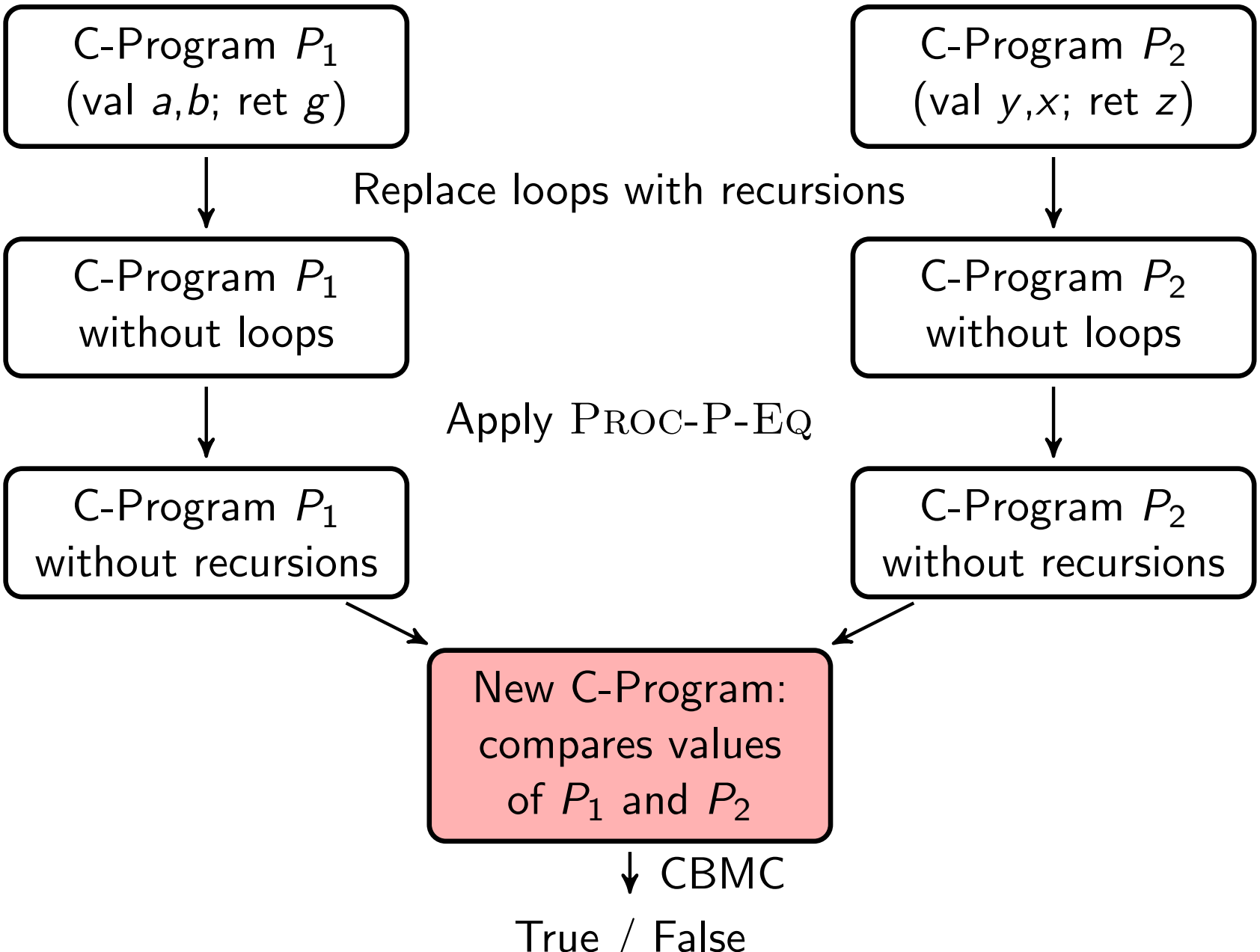
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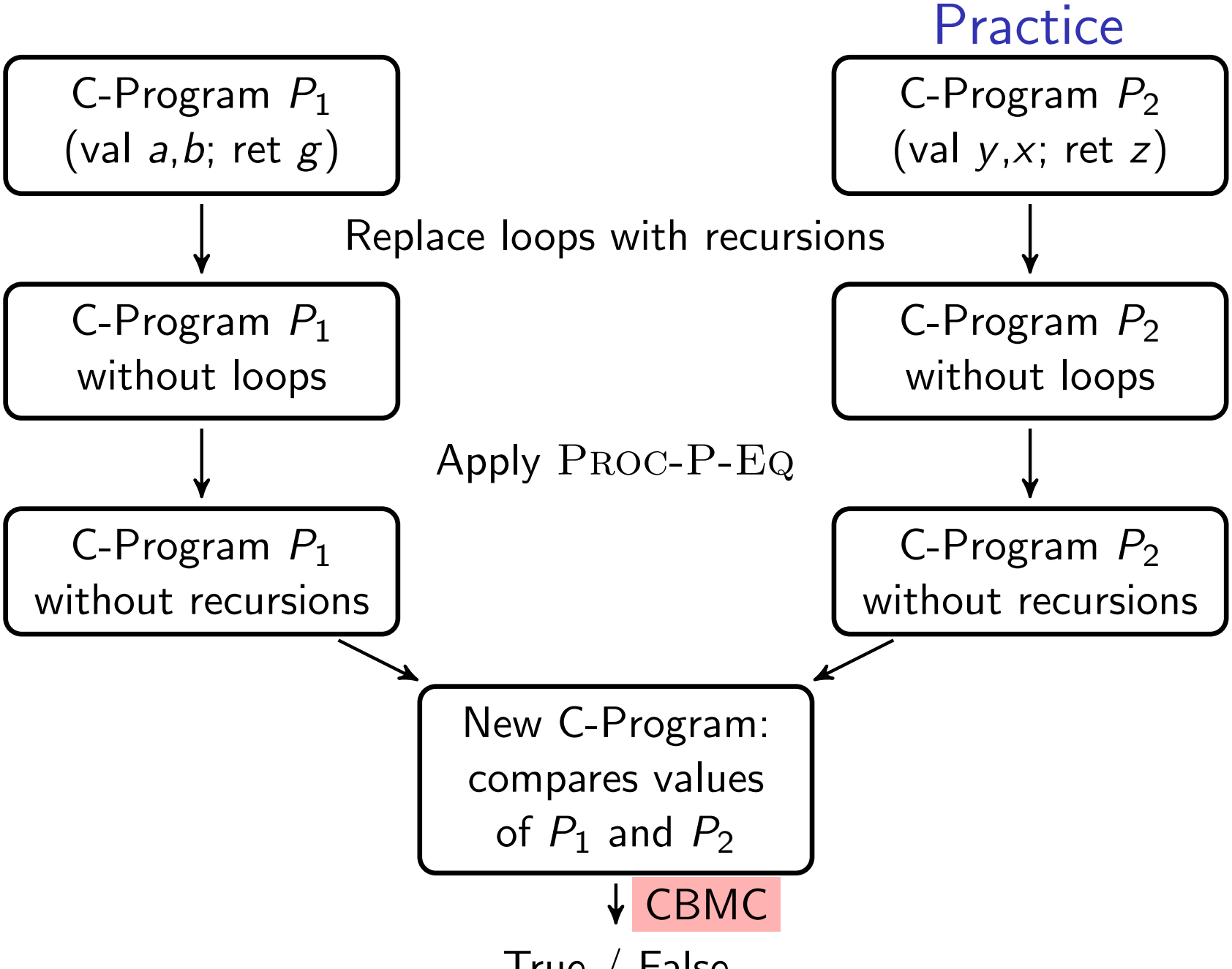
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True / False

Demo

Limitations

Proc-P-Eq

Proc-P-Eq cannot prove recursions where

- procedures are called with different arguments:

```
procedure F                    procedure G
(val n; ret r):                (val n; ret r):
  if n ≤ 1 then                  if n ≤ 1 then
    r := n                         r := n
  else                           else
    call F( n-1 ; r );             call G( n-2 ; r );
    r := n + r                     r := n+(n−1)+r
  fi                             fi
  return                         return
```

- the procedure body is not equivalent
- the Uninterpreted Procedure is too weak

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procedure G
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  else
    call G(n−1; r);
    if r ≥ 0 then r := n+r
    fi
  fi
  return
```

Limitations

Regression Verification Tool

- Condition of equality cannot be specified
- Counterexample not quickly found because of function inlining
- Mapping only by function names and locations

Conclusion

Regression Verification

- Better chance of being adopted than Functional Verification
- More powerful than Regression Testing
- Simple rule $\textsc{Proc-P-Eq}$ for many cases, but not all
- Regression Verification has recently been extended to multi-threaded programs